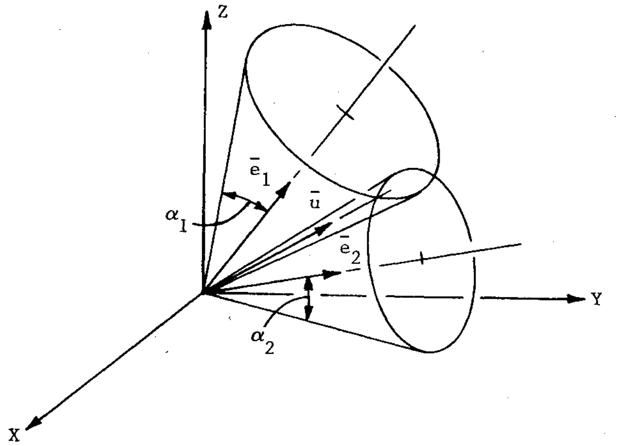

Fig. 1 Geometry.

vector $\bar{u}$ are measured as α_1, α_2, respectively. The problem is to determine $\bar{u}$ (Fig. 1).

Since $\bar{e}_1$, $\bar{e}_2$ are not parallel, a system of base unit vectors consisting of $\bar{e}_1$, $\bar{e}_2$ and $\bar{n}=(\bar{e}_1\times\bar{e}_2)\ /\ |\bar{e}_1\times\bar{e}_2|$ can be introduced. Thus, let $\bar{u}$ be expressed as

$$\bar{u}=\bar{e}_1x+\bar{e}_2y+\bar{n}z \tag{1}$$

where x, y, z are unknowns.

From Eq. (1) and the given data

$$\bar{u}\cdot\bar{e}_1=x+y\ \cos\theta=\cos\alpha_1 \tag{2a}$$

$$\bar{u}\cdot\bar{e}_2=x\ \cos\theta+y=\cos\alpha_2 \tag{2b}$$

where $\cos\theta=\bar{e}_1\cdot\bar{e}_2$, and θ is the angle between $\bar{e}_1$ and $\bar{e}_2$.

Solving Eqs. (2) for x, y

$$x=[\sin^2\theta]^{-1}(\cos\alpha_1-\cos\theta\cos\alpha_2) \tag{3a}$$

$$y=[\sin^2\theta]^{-1}(\cos\alpha_2-\cos\theta\cos\alpha_1) \tag{3b}$$

The $\bar{n}$ component, z, is determined from $\bar{u}\cdot\bar{u}=1$. This gives

$$z=\pm(1-x^2-y^2-2xy\cos\theta)^{1/2} \tag{4a}$$

An equivalent expression is

$$z=\pm(1-x\cos\alpha_1-y\cos\alpha_2)^{1/2} \tag{4b}$$

There are two solutions for z corresponding to $\bar{u}$ lying on one side or the other of the $\bar{e}_1$, $\bar{e}_2$ plane. Additional information is required to resolve this ambiguity. The general solution to the intersection problem is thus given by Eq. (1) with x,y, determined by Eqs. (3) and z by Eq. (4a) or (4b). Observe that for calculation purposes, $\bar{e}_1$, $\bar{e}_2$ and $\bar{n}$ are ultimately expressed in terms of some orthogonal inertial unit vectors.

The solution for z can be verified for the special case where $\bar{u}$ happens to be coplanar with $\bar{e}_1$, $\bar{e}_2$. This can occur in the following circumstances: 1) cones are externally tangent, in which case $\theta=\alpha_1+\alpha_2$; and 2) cones are internally tangent, in which case a) $\theta=\alpha_1-\alpha_2>0$, or b) $\theta=\alpha_2-\alpha_1>0$. For all cases, the solutions can be expressed as

$$x=\pm\frac{\sin\alpha_2}{\sin(\alpha_1\pm\alpha_2)}$$

$$y=\pm\frac{\sin\alpha_1}{\sin(\alpha_2\pm\alpha_1)}$$

$$z=0$$

where the + signs are used for case (1), and the minus signs for both cases (2).

A Simple Scale Length for Shocks about Transverse Gas Jets

D. W. Harvey*
McDonnell Douglas Astronautics Company, Huntington Beach, Calif.

Nomenclature

E = rms error
R = scale length; radius of shock-correlating sphere or cylinder (Fig. 1)
x,y = coordinate system of Ref. 2 (see Fig. 1)
X, Y = nozzle-centered coordinate system (Fig. 1)
$X_{\rm ref}$ = location of shock-correlating sphere or cylinder (Fig. 1)

WHEN a gas jet is injected transversely into a supersonic stream, one of the most prominent and easily measured features produced is the shock about the jet. This shock characterizes the interaction between the jet and the stream. The physical scale of the interaction is of substantial interest. This Note describes a simple method of deriving a scale length from experimental shock shapes.

The method is an extension of that reported by Kallis.[1] It is based on the assumption that the shock about a jet is sufficiently similar to that about a sphere or cylinder so that it can be characterized by a single scale length, the radius of the sphere or cylinder. Although not necessarily intuitively appealing, this assumption is a good one, as will be discussed below.

The essence of the method is that the shock shape expressions developed by Billig[2] are fit, in a least-squares sense, to the experimental shock shape data. Since neither the streamwise location nor the scale length R is known, this fit is a two-parameter one. It is done by assuming a location $X_{\rm ref}$ (see Fig. 1) and calculating a scale length R_i for each data point using Billig's shock shapes, which are of the form $X=f(y,R)$, inverted to give $R=g(x,y)$. The root mean square error is then

$$E(X_{\rm ref})=\frac{1}{N}\left[\sum_{i=1}^{N}\left(\frac{R_i}{\langle R\rangle}-1\right)^2\right]^{1/2}$$

where $\langle R\rangle$ is the average over all N data points. It remains only to minimize E by proper (e.g., Newton-Raphson) adjustment of R. When this is done, $\langle R\rangle=R$, the desired scale length. The extension beyond Kallis' work[1] consists of determining $X_{\rm ref}$ from the fit, rather than directly estimating it from the photographs. The latter method requires visual extrapolation and is believed to induce unnecessary error.

Data from 26 shock-tunnel tests have been analyzed. Test conditions are given by Harvey et al.[3] The slot nozzles used are of 10-to-1 aspect ratio, transverse to the stream. Some results for a typical run are shown in Figs. 2 and 3. In Fig. 2, the actual points obtained from the shock are shown. They extend a considerable distance downstream and thus provide a reasonably critical test of the effect of obstacle shape. This downstream distance is typical for all runs. For each point, the calculated values of R_i are shown for three values of $X_{\rm ref}$. The central one of these three values is the minimum error value. It is clear that, for this value of $X_{\rm ref}$, there is no systematic variation of R with downstream distance; this

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*Senior Engineer/Scientist.

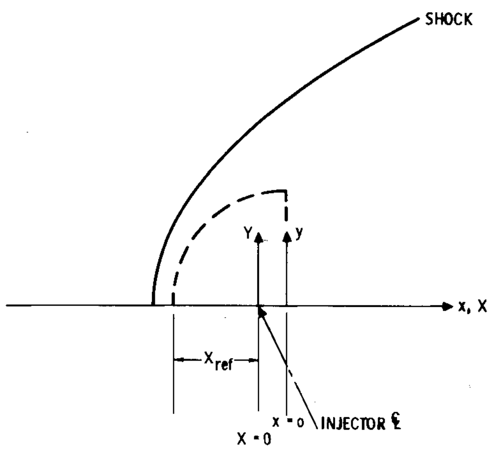

Fig. 1 Geometry for shock shape correlation.

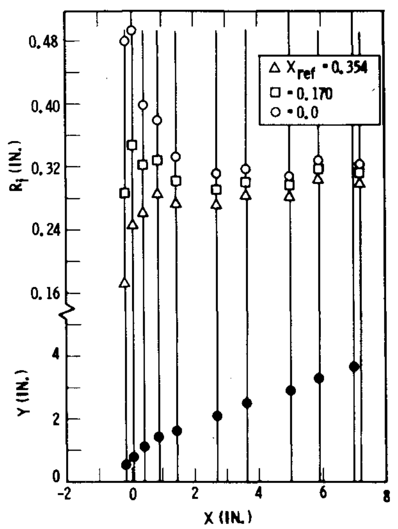

Fig. 2 Sample shock shape data and results.

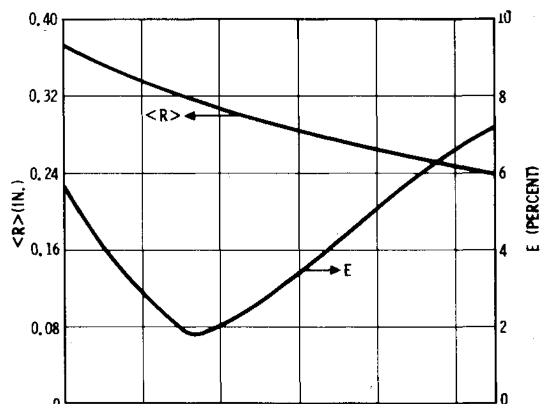

Fig. 3 Results of scale length calculation.

tends to support the shock shape assumption that is the basis of the present method.

Such support can also be found if the errors in the fitting process are small. Figure 3 shows the behavior of rms error E and average scale length $\langle R \rangle$ over part of the range of the values of X_{ref} for which calculations were made, for the same run as the foregoing. The rather sharp minimum shown by E is very satisfactory, and the resulting uncertainty in choice of R (that value of $\langle R \rangle$ for which $E = \min$; here $R = 0.312$) is small. The corresponding value of E is also significantly small, at 1.8%. This is a typical value; the largest value of E among all runs is 5.6%, and the average is 1.9%. No significant difference exists between fuel sources or nozzles. The average value of E over slot nozzle tests, for example, is 1.7%.

These values are small enough to confirm the original assumption that the experimental shock shapes can be well represented by Billig's shock shape expressions.[2] (However, caution should be observed in using the cylinder shock on data from nozzles of transverse aspect ratio less than 10, or using the sphere shock on other than circular nozzles.)

Acknowledgment

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Effect of Surface Roughness on Blunt Body Boundary-Layer Transition

A. J. Laderman*
Aeronutronic Ford Corporation, Newport Beach, Calif.

Nomenclature

D	= model diameter
k	= average height of roughness elements
M	= Mach number
p	= pressure
$Re_{\infty,D}$	$= \rho_\infty u_\infty D/\mu_\infty$
Re_θ	= Reynolds number based on momemtum thickness $= \rho_e u_e \theta/\mu_e$
T	= temperature
u	= velocity
ρ	= density
μ	= viscosity
φ	= angular position of model survey station
Δ	= roughness parameter $kT_e/\theta T_w$
θ	= momentum thickness

Subscripts

e	= boundary-layer edge condition
∞	= freestream condition
w	= wall condition
o	= stagnation condition

Introduction

THIS Note describes the effects of three-dimensional, distributed roughness on boundary-layer transition on a hypersonic blunt body. Although the effects of surface roughness on transition are qualitatively known, relatively little specific information is available in the literature.

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Index categories: Boundary-Layer Stability and Transition; Boundary Layers and Convective Heat Transfer—Laminar.

*Principal Scientist, Fluid Mechanics Section. Member AIAA.